Applied Mathematics : Chapter 8.1 Review Problems   Noah Sassman

8.1.15] If $A = \begin{bmatrix} 4 & 5 \\ -6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 6 \\ 8 & -10 \end{bmatrix}$, Find (a), (b), and (c)

(a) A + B

$$A+B = \begin{bmatrix} (4+(-2)) & (5+6) \\ ((-6)+8) & (9+(-10)) \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 2 & -1 \end{bmatrix}$$

(b) B − A

$$B-A = \begin{bmatrix} ((-2)-4) & (6-5) \\ (8-(-6)) & ((-10)-9) \end{bmatrix} = \begin{bmatrix} -6 & 1 \\ 14 & -19 \end{bmatrix}$$

(c) 2A + 3B

$$2A+3B = 2\begin{bmatrix} 4 & 5 \\ -6 & 9 \end{bmatrix} + 3\begin{bmatrix} -2 & 6 \\ 8 & -10 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 8 & 10 \\ -12 & 18 \end{bmatrix} + \begin{bmatrix} -6 & 18 \\ 24 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} (8+(-6)) & (10+18) \\ ((-12)+24) & (18+(-30)) \end{bmatrix} = \begin{bmatrix} 2 & 28 \\ 12 & -12 \end{bmatrix}$$

8.1.33] If $A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 10 \\ 2 & 5 \end{bmatrix}$, verify the given property, $[AB]^T = B^T A^T$, by computing the left and right members of the given equality.

Left side

$$AB = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 & 10 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(4)+(4)(2) & (2)(10)+(4)(5) \\ (-3)(4)+(2)(2) & (-3)(10)+(2)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 40 \\ -8 & -20 \end{bmatrix}$$

$$\Rightarrow AB^T = \begin{bmatrix} 16 & -8 \\ 40 & -20 \end{bmatrix} \checkmark$$

Right Side

$$B^T = \begin{bmatrix} 4 & 2 \\ 10 & 5 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} (4)(2)+(2)(4) & (4)(-3)+(2)(2) \\ (10)(2)+(5)(4) & (10)(-3)+(5)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -8 \\ 40 & -20 \end{bmatrix} \checkmark$$

8.2 Review Solutions

Alex
Cala

1) $x_1 - x_2 = 11$
$4x_1 + 3x_2 = -5$ $\longrightarrow$ $\left[\begin{array}{cc|c} 1 & -1 & 11 \\ 4 & 3 & -5 \end{array}\right]$

$\underline{R_2 \to R_2 - 4R_1}$

$\left[\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 7 & -49 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{7} R_2} \left[\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 1 & -7 \end{array}\right]$

$\underline{R_1 \to R_1 + R_2}$

$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -7 \end{array}\right] \longrightarrow$ $x_1 = 4$, $x_2 = -7$

$\boxed{(x_1, x_2) = (4, -7)}$

8.3 Rank of a Matrix

Alvaro Rodriguez
05 DEC 2021

Find the rank of the given Matrix

3) $\begin{bmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix} \xrightarrow{\frac{\text{Row 1}}{2}} \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$ $6(R1) - R2$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$

$R1 + R_3 \Rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\boxed{\text{Rank} = 1}$

Find the rank of the given matrix

7) $\begin{bmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{bmatrix}$ $\begin{array}{l} (3)R_1 - R_2 \\ \\ -(7)R1 + R3 \\ -(4)R1 + R4 \end{array}$ $\begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 13 \\ 0 & 13 \end{bmatrix}$ $R_3 - R_4$ $\begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 13 \\ 0 & 0 \end{bmatrix}$

Swap R2 & R3 $\begin{bmatrix} 1 & -2 \\ 0 & 13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\boxed{\text{Rank} = 2}$

8.4) P.8.

$$\begin{bmatrix} 0 & 2 & 4 & 0 \\ 1 & 2 & -2 & 3 \\ 5 & 1 & 0 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$$C_{23} = (-1)^{2+3} = (-1)^5 = -1$$

$$M_{23} = (-1) \begin{vmatrix} 0 & 2 & 0 \\ 5 & 1 & -1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -[0[2+1] - 2[10+1] + 0[5-1]]$$

$$= -(-22)$$

$$M_{23} = 22$$

Section 8.5 Solutions

Brenton Kearney
Math 310-03

1) $$B = \begin{pmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{pmatrix} \xrightarrow{R_2 \cdot \frac{1}{3}} \begin{pmatrix} 2a_1 & a_2 & a_3 \\ 2b_1 & b_2 & b_3 \\ 2c_1 & c_2 & c_3 \end{pmatrix} \xrightarrow{C_1 \cdot \frac{1}{2}} \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Since $|A| = 5$, we can get $|B|$ based on matrix transformations.

When multiplying a row or column by a scalar, you multiply determinant by that same scalar. In this case, since we went backwards from $B \to A$, we multiply by the inverse of those scalars.

So $|B| = 3 \cdot 2 \cdot |A| = 6 \cdot 5 = \boxed{30}$

2) $$\begin{pmatrix} 2 & 4 & 5 \\ 4 & 2 & 0 \\ 8 & 7 & -2 \end{pmatrix} \xrightarrow{R_1 \cdot \frac{1}{2}} \begin{pmatrix} 1 & 2 & 5/2 \\ 4 & 2 & 0 \\ 8 & 7 & -2 \end{pmatrix} \xrightarrow[R_3 - 8R_1]{R_2 - 4R_1} \begin{pmatrix} 1 & 2 & 5/2 \\ 0 & -6 & -10 \\ 0 & -9 & -22 \end{pmatrix} \xrightarrow{R_2 \cdot \frac{1}{2}}$$

$$\to \begin{pmatrix} 1 & 2 & 5/2 \\ 0 & -3 & -5 \\ 0 & -9 & -22 \end{pmatrix} \xrightarrow{R_3 - 3R_2} \begin{pmatrix} 1 & 2 & 5/2 \\ 0 & -3 & -5 \\ 0 & 0 & -7 \end{pmatrix}$$

This is lower triangular, so the determinant equals the product of each factor along the main diagonal and any scalars used to change row values. No rows were swapped.

So $\det = \underbrace{(1 \cdot -3 \cdot -7)}_{\text{diagonal}}\underbrace{(2)(2)}_{\text{scalars}} = (21)(4) = \boxed{84}$

P. 24 | 8.4

$$\begin{bmatrix} 1 & 1 & 1 \\ x & y & z \\ 2+x & 3+y & 4+z \end{bmatrix} = A$$

$$= C_{11} + C_{12} + C_{13} \Rightarrow C_{11}^{1+1} = 1 \quad , \quad C_{12}^{1+2} = -1 \quad , \quad C_{13}^{1+3} = 1$$

$$|C_{11}| = 1 \cdot \begin{vmatrix} y & z \\ 3+y & 4+z \end{vmatrix} \Rightarrow 1(y[4+z] - z[3+y])$$

$$\Rightarrow 4y - 3z$$

$$|C_{12}| = -1 \begin{vmatrix} x & z \\ 2+x & 4+z \end{vmatrix} \Rightarrow -1[4x + xz - 2z - xz]$$

$$\Rightarrow -4x + 2z$$

$$|C_{13}| = 1 \begin{vmatrix} x & y \\ 2+x & 3+y \end{vmatrix} \Rightarrow 1[x(3+y) - y(2+x)]$$

$$\Rightarrow 3x - 2y$$

$$\det A = 4y - 3z - 4x + 2z + 3x - 2y$$

$$= -x + 2y - z$$

Caleb McCrillis

8.6 Inverse of a Matrix

SOLUTIONS

1

a. $\begin{bmatrix} 2 & -1 & 5 \\ 3 & 0 & -2 \\ 1 & 4 & 0 \end{bmatrix}$

$$\det A: \begin{vmatrix} 2 & -1 & 5 \\ 3 & 0 & -2 \\ 1 & 4 & 0 \end{vmatrix} = 2\begin{vmatrix} 0 & -2 \\ 4 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} + 5\begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 16 + 2 + 60$$

$$\det A = 78$$

Cofactor of A

$$\begin{bmatrix} \begin{vmatrix} 0 & -2 \\ 4 & 0 \end{vmatrix} & \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} -1 & 5 \\ 4 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} -1 & 5 \\ 0 & -2 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 8 & -2 & 12 \\ 20 & -5 & -9 \\ 2 & 19 & 3 \end{bmatrix}$$

Adjoint of A

cofactor A^T

$$\begin{bmatrix} 8 & 20 & 2 \\ -2 & -5 & 19 \\ 12 & -9 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{\det A}$$

$$\frac{1}{78} \cdot \begin{bmatrix} 8 & 20 & 2 \\ -2 & -5 & 19 \\ 12 & -9 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{39} & -\frac{10}{39} & \frac{1}{39} \\ -1/39 & \frac{-5}{78} & 19/78 \\ 2/13 & -3/26 & \frac{1}{26} \end{bmatrix}$$

SOLUTIONS

2. $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ 0 \\ 6 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 5 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -r_1 \\ -5r_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & -5 & -5 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ +r_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -6 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \cdot -\frac{1}{5} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{6}{5} & -\frac{1}{5} & -\frac{1}{5} \end{array}\right] \begin{array}{l} -r_3 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{6}{5} & -\frac{1}{5} & -\frac{1}{5} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ -1 & 1 & 0 \\ \frac{6}{5} & -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}$$

$X = A^{-1} B$

$$X = \begin{bmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ -1 & 1 & 0 \\ \frac{6}{5} & -\frac{1}{5} & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5}(-4) + \frac{1}{5}(0) + \frac{1}{5}(6) \\ -1(-4) + 1(0) + 0(6) \\ \frac{6}{5}(-4) - \frac{1}{5}(0) - \frac{1}{5}(6) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$x_1 = 2 \quad x_2 = 4 \quad x_3 = 6$

8.6
1 sol.

Cameron Marshall

$$A = \begin{pmatrix} 1 & 1/2 \\ 2 & 3/2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

$$AB = BA = I$$

$$AB = \begin{bmatrix} 3-2 & -1+1 \\ 6-6 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} 3-2 & 3/2-3/2 \\ -4+4 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\boxed{\text{Verified } \checkmark}$$

8.6
#7 sol.

Cameron Marshall

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 4 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\det A = |A| = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$|A| = 1(4+4) - 3(2-4) + 5(-2-4)$$

$$|A| = -16 \neq 0$$

Non-Singular

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$

$C_{11} = \begin{vmatrix} 4 & 4 \\ -1 & 1 \end{vmatrix} = 8$ $\quad C_{21} = -8$ $\quad C_{31} = -8$

$C_{12} = -\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = 2$ $\quad C_{22} = -4$ $\quad C_{32} = 6$

$C_{13} = \begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} = -6$ $\quad C_{23} = 4$ $\quad C_{33} = -2$

$$A^{-1} = \frac{1}{-16} \begin{pmatrix} 8 & -8 & -8 \\ 2 & -4 & 6 \\ -6 & 4 & -2 \end{pmatrix}$$

$$\boxed{A^{-1} = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ -1/8 & 1/4 & -3/8 \\ 3/8 & -1/4 & 1/8 \end{pmatrix}}$$

8.7: Cramer's Rule (solutions) MATH-310-03 Mia Tabladillo

10) $4x + 3y + 2z = 8$
$-x + 2z = 12$
$3x + 2y + z = 3$

$\det A = \begin{pmatrix} 4 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix} = 1$ $\det A_1 = \begin{pmatrix} 8 & 3 & 2 \\ 12 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix} = -2$

$\det A_2 = \begin{pmatrix} 4 & 8 & 2 \\ -1 & 12 & 2 \\ 3 & 3 & 1 \end{pmatrix} = 2$ $\det A_3 = \begin{pmatrix} 4 & 3 & 8 \\ -1 & 0 & 12 \\ 3 & 2 & 3 \end{pmatrix} = 5$

$\boxed{\begin{array}{l} x_1 = -2 \\ x_2 = 2 \\ x_3 = 5 \end{array}}$

$x_1 = \frac{-2}{1}$ $x_2 = \frac{2}{1}$ $x_3 = \frac{5}{1}$

13) $(\cos 25°)T_1 - (\cos 15°)T_2 = 0$
$(\sin 25°)T_1 + (\sin 15°)T_2 = 300$

$\det A = \begin{pmatrix} \cos 25° & -\cos 15° \\ \sin 25° & \sin 15° \end{pmatrix} = 0.6428$ $\det A_1 = \begin{pmatrix} 0 & -\cos 15° \\ 300 & \sin 15° \end{pmatrix} = 289.7777$

$\det A_2 = \begin{pmatrix} \cos 25° & 0 \\ \sin 25° & 300 \end{pmatrix} = 271.8923$

$\boxed{x_1 = \frac{289.7777}{0.6428} \qquad x_2 = \frac{271.8923}{0.6428}}$

8.8 - Finding eigenvalues & characteristic polynomials

Daniel Rosales

1) Remember that $AK = \lambda K$

$$\begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2+\sqrt{2} \\ 2 \end{bmatrix} = \begin{bmatrix} 2+2\sqrt{2} \\ 2\sqrt{2} \end{bmatrix} = \underline{\lambda} \begin{bmatrix} 2+\sqrt{2} \\ 2 \end{bmatrix}$$

K is an eigenvector with corresponding eigenvalue $\lambda = \sqrt{2}$.

2) solve using $\det(A - \lambda I_3)$ and find roots

$$\Rightarrow \begin{vmatrix} 1-\lambda & 6 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} - 6\begin{vmatrix} 0 & 1 \\ 0 & 2-\lambda \end{vmatrix} + 0$$

Solving along the top row, we cancel one of the rows with a zero.

$$= (1-\lambda)(\lambda^2 - 4\lambda + 3) - 6(0) = (1-\lambda)(1-\lambda)(3-\lambda)$$

$$= (1-\lambda)^2(3-\lambda) = 0$$

A has eigenvalues of $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 3$

since none of them are zero, A is nonsingular.

Recall $A^{-1}K = \frac{1}{\lambda}K$, so A^{-1} has eigenvalues of $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = \frac{1}{3}$

Section 2.2 Solutions (Problem #8)

Ian Baker

8. $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

remember: $e^{-2x-y} = e^{-2x}e^{-y}$

$e^x y \frac{dy}{dx} = e^{-y} + e^{-2x}e^{-y} \Rightarrow e^x y \frac{dy}{dx} = e^{-y}(1+e^{-2x}) \Rightarrow$

Separate variables: $\frac{y}{e^{-y}}dy = \frac{1+e^{-2x}}{e^x}dx \Rightarrow ye^y dy = \left(\frac{1}{e^x} + \frac{e^{-2x}}{e^x}\right)dx$

simplify

$\Rightarrow \int ye^y dy = \int (e^{-x} + e^{-3x})dx$

First: $\int ye^y dy$ $\quad u = y \quad dv = e^y dy$ $\quad du = dy \quad v = e^y$

$= uv - \int v\,du = ye^y - \int e^y dy = ye^y - e^y + C$

Second: $\int e^{-x} + e^{-3x} dx$

remember: $\int e^{\pm ax} dx = \pm\frac{1}{a}e^{ac} + C$

$\therefore \int e^{-x} + e^{-3x} dx = -e^{-x} - \frac{1}{3}e^{-3x} + C$

$\boxed{\text{Answer: } ye^y - e^y = -e^{-x} - \frac{1}{3}e^{-3x} + C}$

Section 2.2 Solutions (Problem #24)

Ian Baker

24. $\frac{dy}{dx} = \frac{y^2-1}{x^2-1}$, $y(2)=2$

multiply both sides by $\frac{dx}{y^2-1}$: $\frac{dy}{y^2-1} = \frac{dx}{x^2-1} \rightarrow \int \frac{dy}{y^2-1} = \int \frac{dx}{x^2-1}$

$\therefore \int \frac{dy}{(y+1)(y-1)} = \int \frac{dx}{(x+1)(x-1)}$

$\frac{1}{(y+1)(y-1)} = \frac{A}{y+1} + \frac{B}{y-1}$ plus in $y=-1$: $A = -\frac{1}{2}$
plus in $y=1$: $B = \frac{1}{2}$

$\therefore \frac{-\frac{1}{2}}{y+1} + \frac{\frac{1}{2}}{y-1}$ do the same for x to obtain: $\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$

$$\int \left(\frac{-\frac{1}{2}}{y+1} + \frac{\frac{1}{2}}{y-1}\right) dy = \int \left(\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\right) dx$$

$$-\frac{1}{2}\ln|y+1| + \frac{1}{2}\ln|y-1| = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C_1$$

multiply by 2: $-\ln|y+1| + \ln|y-1| = -\ln|x+1| + \ln|x-1| + \ln C_1$

$$\Rightarrow e^{\ln\left(\frac{y-1}{y+1}\right)} = e^{\ln\left(\frac{C_1(x-1)}{x+1}\right)}$$

$\frac{y-1}{y+1} = \frac{C_1(x-1)}{x+1}$ $y(2)=2 \Rightarrow \frac{2-1}{2+1} = \frac{C_1(2-1)}{2+1}$

$2-1 = C_1(2-1)$

$C(1) = 1$

$C_1 = 1$

$\therefore$ implicit $= \boxed{\frac{y-1}{y+1} = \frac{x-1}{x+1}}$

explicit: simplify implicit

$\Rightarrow (y-1)(x+1) = (y+1)(x-1)$

$\rightarrow \cancel{xy} - x + y - \cancel{1} = \cancel{xy} + x - y - \cancel{1}$

$y - x = x - y$

$2x = 2y$

$\boxed{y = x}$ explicit

2.2 Seperable Equations

Honorebel S. Walker
7 December 2021

Solve the Given Differential Equation by Sepperation of Variables

#3 $dx + e^{3x}dy = 0$

$$dx + e^{3x}dy = 0$$

$$dx = \frac{-e^{3x}dy}{-e^{3x}}$$

$$\frac{-1}{-e^{3x}}dx = \frac{-e^{3x}}{-e^{3x}}dy$$

$$-e^{-3x}dx = dy$$

$$\left[\int e^{ax}dx = \frac{e^{ax}}{a} + C\right]$$

$$\int dy = -\int e^{-3x}dx$$

$$y = -\left(\frac{e^{-3x}}{-3}\right) + C$$

$$= y = \frac{e^{-3x}}{3} + C$$

$$\boxed{y = \frac{e^{-3x}}{3} + C}$$

#6 $\frac{dy}{dx} + 2xy^2 = 0$

$$\frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{y^2} = -2xdx$$

$$\int \frac{dy}{y^2} = -2\int xdx$$

$$\frac{-1}{y} = -2\frac{x^2}{2} + C$$

$$-\frac{1}{y} = -x^2 + C$$

$$\frac{1}{y} = x^2 - C$$

$$y = \frac{1}{x^2 - C}$$

$$\boxed{y = \frac{1}{x^2 - C}}$$

2.3 Solutions

General form:

$$\frac{dy}{dx} + p(x)y = f(x)$$

$$I.F. = e^{\int p\,dx}$$

1) $\dfrac{dy}{dx} + y = e^{3x}$

$$I.F. = e^{\int 1\,dx} = e^{x}$$

$$ye^{x} = \int e^{3x} e^{x}\,dx + c$$

$$= \int e^{4x}\,dx + c$$

$$ye^{x} = \frac{e^{4x}}{4} + c$$

$$y = \frac{e^{3x}}{4} + ce^{-x} \text{ on } -\infty < x < \infty$$

2) $(x+2)^2 \dfrac{dy}{dx} = 5 - 8y - 4xy$

$$= 5 - 4y(x+2)$$

$$\frac{(x+2)^2 \frac{dy}{dx} + 4y(x+2)}{(x+2)^2} = \frac{5}{(x+2)^2}$$

$$\frac{dy}{dx} + \frac{4y}{x+2} = \frac{5}{(x+2)^2}$$

$$IF = e^{\int \frac{4}{x+2}dx} = e^{4\int \frac{1}{x+2}dx}$$

$$= e^{4\ln(x+2)} = (x+2)^4$$

$$y(x+2) = \int \frac{5}{(x+2)^2}(x+2)^4\,dx + c$$

$$= \int 5(x+2)^2\,dx + c$$

$$= \frac{5}{3}(x+2)^3 + c$$

$$y = \frac{5}{3}(x+2)^{-1} + c(x+2)^{-4} \quad \text{for all } x > -2$$

John Cizin
Section 2.3

5) $y' + 3x^2 y = x^2$

Standard form: $\frac{dy}{dx} + P(x)y = f(x)$

$y' = \frac{dy}{dx} \rightarrow \frac{dy}{dx} + 3x^2 y = x^2$

$P(x) = 3x^2$

$f(x) = x^2$

$e^{\int P(x)\,dx} = e^{\int 3x^2 dx} = e^{3\int x^2 dx} = e^{\cancel{3}\frac{1}{\cancel{3}}x^3} = e^{x^3}$

$e^{x^3} \cdot \left(\frac{dy}{dx} + 3x^2 y\right) = x^2 \cdot e^{x^3} = e^{x^3}\frac{dy}{dx} + 3x^2 e^{x^3} y = x^2 e^{x^3}$

$= e^{x^3}\frac{dy}{dx} + y\frac{d}{dx}\left(e^{x^3}\right) = x^2 e^{x^3}$

$= \frac{d}{dx}\left(e^{x^3} y\right) = x^2 e^{x^3}$

$\int \cancel{\frac{d}{dx}}\left(e^{x^3} y\right) = \int \left(x^2 e^{x^3}\right) dx$

$e^{x^3} y = \int x^2 e^{x^3} dx$

$e^{x^3} y = \frac{1}{3}\int e^u du$

$x^3 = u$

$3x^2 dx = du$

$x^2 dx = \frac{1}{3} du$

$e^{x^3} y = \frac{1}{3}e^u + c$

$e^{x^3} y = \frac{1}{3}e^{x^3} + c$

$y = \frac{1}{3}\cancel{e^{x^3}}\cancel{e^{-x^3}} + ce^{-x^3}$

$\boxed{y = \frac{1}{3} + ce^{-x^3}} \longrightarrow$ transient terms?

$y_c(x) = ce^{-x^3}$ as $x \rightarrow \infty = 0$

is negligible so it is a transient term.

John Cizin
Section 2.3

5.) Find the general solution of the given differential equation. Determine whether there are any transient terms in the general solution.

$$y' + 3x^2 y = x^2$$

18.) Find the general solution of the given differential equation.

$$\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x) y = 1$$

John Cizin
Section 2.3

18.) $\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x) y = 1$

divide by $\cos^2 x \sin x$

$$\frac{dy}{dx} + \frac{\cos^{\cancel{3}} x}{\cancel{\cos^2 x} \sin x} y = \frac{1}{\cos^2 x \sin x}$$

$$\frac{dy}{dx} + \cot x\, y = \frac{1}{\cos^2 x \sin x}$$

$$P(x) = \cot x$$

$$f(x) = \frac{1}{\cos^2 x \sin x}$$

$$\text{I.F.} = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$$

equation $\rightarrow$ $y \cdot IF = \int f(x) \cdot IF\, dx + c$

$$y \cdot \sin x = \int \frac{1}{\cos^2 x \sin x} \cdot \sin x\, dx + c$$

$$y \sin x = \int \sec^2 x\, dx + c$$

$$\frac{y \cancel{\sin x}}{\cancel{\sin x}} = \frac{\tan x + c}{\sin x}$$

$$y = \sec x + c \frac{1}{\sin x}$$

$$y = \sec x + c \cdot \csc x \quad 0 < x < \frac{\pi}{2}$$

$$(3x^2y+e^y)\,dx+(x^3+xe^y-2y)\,dy=0$$

$$\frac{\partial M}{\partial Y}=3x^2+e^y \qquad \frac{\partial N}{\partial X}=3x^2+e^y \qquad \text{exact}$$

$$\frac{\partial f}{\partial x}=3x^2y+e^y+h'(y))$$

$$\int\frac{\partial f}{\partial x}=f=\int 3x^2y+e^y+h'(y)$$

$$\int u\,dv=uv-\int v\,du$$
$$u=y \quad v=x^3$$
$$du=1 \quad dv=3x^2$$

$$=yx^3-\int x^3+\int e^y+\int h'(y)$$

$$=x^3y-\frac{x^4}{4}+e^y+h(y)$$

$$\frac{\partial f}{\partial y}=x^3+xe^y\underbrace{-2y}_{h'(y)}$$

$$\int h'(y)=\int -2y=-2\frac{y^2}{2}=-y^2$$

$$\therefore\ f(x,y)=x^3y-\frac{x^4}{4}+e^y-y^2$$

$$\therefore\ x^3y-\frac{x^4}{4}+e^y-y^2=C \quad \text{is solution}$$

$$(2y\sin x\cos x - y + 2y^2 e^{xy^2})\,dx = (x - \sin^2 x - 4xye^{xy^2})\,dy$$

$$\underbrace{(2y\sin(x)\cos(x) - y) + 2y^2 e^{xy^2})\,dx}_{N(x,y)\,dx} + \underbrace{(-x + \sin^2(x) + 4xye^{xy^2})\,dy}_{M(x,y)\,dy} = 0$$

$$\frac{\partial N}{\partial y} = 2\sin(x)\cos(x) - 1 + 4ye^{xy^2} + 2xye^{xy^2}(2y^2)$$

$$= 2\sin(x)\cos(x) - 1 + 4ye^{xy^2} + 4xy^3 e^{xy^2}$$

$$\frac{\partial M}{\partial x} = -1 + 2\sin(x)\cos(x) + 4ye^{xy^2} + 4xy \cdot y^2 e^{xy^2}$$

$$= 2\sin(x)\cos(x) + 4ye^{-xy^2} - 1 + 4xy^3 e^{xy^2}$$

$$\frac{\partial f}{\partial x} = 2y\sin(x)\cos(x) - y + 2y^2 e^{xy^2}$$

$$\int \frac{\partial f}{\partial x} = f = 2y(-\cos(x))\sin(x) - yx + 2y^2 \frac{e^{xy^2}}{y^2} + h(y)$$

$$= 2y\frac{\sin^2 x}{2} - xy + 2y^2 \frac{e^{xy^2}}{y^2} + h(y)$$

$$= y\sin^2 x - xy + 2e^{xy^2} + h(y)$$

$$\frac{\partial f}{\partial y} = y\sin^2 x - xy + 2e^{xy^2} + h(y)$$

$$= \sin^2 x - x + 2e^{xy^2}(2xy) + h'(y)$$

$$= \sin^2 x - x + 4xye^{xy^2} + h'(y)$$

$$h'(y) = 0$$

$$h(y) = 0$$

$$f(x,y) = y\sin^2 x - xy + 2e^{xy^2} + C$$

2.4 Making exact

Juliana Grigg

solutions

30) $\underbrace{(x^2+2xy-y^2)}_{\frac{\partial}{\partial y}=2x-2y}dx+\underbrace{(y^2+2xy-x^2)}_{\frac{\partial}{\partial x}=2y-2x}dy=0$ If: $\mu(x,y)=(x+y)^{-2}$

$\frac{\partial}{\partial x}=2y-2x$
$=-2x+2y$ ← not exact

$\mu(x,y)=(x+y)^{-2}$

$[(x^2+2xy-y^2)dx+(y^2+2xy-x^2)dy]\cdot(x+y)^{-2}=0$

$\frac{(x^2+2xy-y^2)}{(x+y)^2}dx+\frac{(y^2+2xy-x^2)}{(x+y)^2}dy=0$

$\frac{\partial}{\partial y}M(x,y)=\frac{\partial}{\partial y}\frac{(x^2+2xy-y^2)}{(x+y)^2}=\frac{(x+y)^2\cdot(2x-2y)-(x^2+2xy-y^2)\cdot2(x+y)}{(x+y)^4}$

$=\frac{\cancel{(x+y)}[(x+y)(2x-2y)-2(x^2+2xy-y^2)]}{(x+y)^{\cancel{4}\,3}}$

$=\frac{(x+y)(2x-2y)-2(x^2+2xy-y^2)}{(x+y)^3}$

$=\cancel{2x^2}-\cancel{2xy}+\cancel{2xy}-2y^2-\cancel{2x^2}-4xy+\cancel{2y^2}$

$=\frac{-4xy}{(x+y)^3}$

$\frac{\partial}{\partial x}N(x,y)=\frac{\partial}{\partial x}\frac{(y^2+2xy-x^2)}{(x+y)^2}=\frac{(x+y)^2\cdot(2y-2x)-(y^2+2xy-x^2)\cdot2(x+y)}{(x+y)^4}$

$=\frac{(x+y)(2y-2x)-2(y^2+2xy-x^2)}{(x+y)^3}$

$=\frac{\cancel{2xy}-\cancel{2x^2}+\cancel{2y^2}-\cancel{2xy}-\cancel{2y^2}-4xy+\cancel{2x^2}}{(x+y)^3}$

$=\frac{-4xy}{(x+y)^3}$

$F(x,y)=\int M(x,y)+h(y)$

$=\int\frac{x^2+2xy-y^2}{(x+y)^2}dx+h(y)$

$=\int\frac{(x+y)^2-2y^2}{(x+y)^2}dx$

Juliana
Griggs

$$= \int 1 - \frac{2y^2}{(x+y)^2}\,dx$$

$$= x - 2y^2 \int \frac{1}{(x+y)^2}\,dx$$

$$u = (x+y)$$
$$du = dx$$

$$-2y^2 \int u^{-2}\,du$$

$$-2y^2(-u^{-1})$$

$$F(x,y) = x + \frac{2y^2}{x+y} + h(y)$$

$$N(x,y) = \frac{\partial}{\partial y}F(x,y) = \frac{\partial}{\partial y}\left(x + \frac{2y^2}{x+y} + h(y)\right) = \frac{(y^2+2xy-x^2)}{(x+y)^2}$$

$$\frac{(x+y)\cdot 4y - 2y^2}{(x+y)^2} + h'(y) = \frac{(y^2+2xy-x^2)}{(x+y)^2}$$

$$\frac{4xy+4y^2-2y^2}{(x+y)^2} + h'(y) = \frac{(y^2+2xy-x^2)}{(x+y)^2}$$

$$h'(y) = \frac{\cancel{y^2}+2xy-x^2-4xy-4y^2\cancel{+2y^2}}{(x+y)^2}$$

$$h'(y) = \frac{-y^2-2xy-x^2}{(x+y)^2}$$

$$= \frac{-(x^2+2xy+y^2)}{(x+y)^2}$$

$$= -1$$

$$h(y) = -y$$

$$F(x,y) = x + \frac{2y^2}{x+y} - y$$

$$= \frac{x(x+y)+2y^2-y(x+y)}{(x+y)}$$

$$= \frac{x^2+\cancel{xy}+2y^2-\cancel{xy}-y^2}{(x+y)}$$

$$\boxed{\frac{x^2+y^2}{x+y} = C}$$

Juliana grigg

34) $\underbrace{\cos x\,dx}_{\frac{\partial}{\partial y}=0} + \underbrace{\left(1+\frac{2}{y}\right)\sin x\,dy}_{\sin x + \frac{2\sin x}{y}} = 0$

$\frac{\partial}{\partial x} = \cos x + \frac{2}{y}\cos x$

$\frac{N_x - M_y}{M} = \frac{\cos x + \frac{2}{y}\cos x}{\cos x} = 1 + \frac{2}{y}$ ← Fxn of y

IF : $e^{\int 1+\frac{2}{y}\,dy}$

$= e^{y + 2\ln y}$

$= e^y e^{\ln y^2}$

$= y^2 e^y$

$(\cos x\,dx + (\sin x + \frac{2\sin x}{y})dy = 0)\,y^2e^y$

$(y^2e^y\cos x)dx + (y^2e^y\sin x + 2ye^y\sin x)dy = 0$

$f(x,y) = \int M(x,y)\,dx + h(y)$

$= \int y^2e^y\cos x\,dx + h(y)$

$= y^2e^y\sin x + h(y)$

$N(x,y) = \frac{\partial}{\partial y}f(x,y) = \frac{\partial}{\partial y}(y^2e^y\sin x + h(y))$

$= \sin x(y^2\cdot e^y + e^y\cdot 2y) + h'(y)$

$= \sin x(y^2e^y + 2ye^y) + h'(y) = y^2e^y\sin x + 2ye^y\sin x$

$h'(y) = y^2\cancel{e^y\sin x} + 2y\cancel{e^y\sin x} - y^2e^y\sin x - 2ye^y\sin x$

$h'(y) = 0$

$h(y) = C$

$f(x,y) = y^2e^y\sin x + C$

$\boxed{C = y^2e^y\sin x}$

Katie Long 2.6: Euler's method # 1

Use Euler's method to obtain a four decimal aproximation of the indicated value. Carry out the recursion of (3) by hand using $h = 0.1$ and then using $h = 0.05$

$y' = 2x - 3y + 1$ $y(1) = 5$; $y(1.2)$

Katie Long

2.6 : Euler's Method

#8

Use a numerical solver and Eulers method to obtain a four-decimal approximation of the indicated value. First use $h = 0.1$ and then use $h = 0.05$

$$y' = xy + \sqrt{y} \qquad y(0) = 1; \quad y(0.5)$$

Kate Jordan - Section I

2.7

Solution #1: $P = P_0 e^{kt}$

$P(10) = 500 + (500 \times .15)$

$= 575$

$575 = 500e^{10k}$

$k \approx 0.014$

at $t = 30 \rightarrow P(30) = 500e^{0.014(30)}$

$= \boxed{760}$ people

rate at $t = 30 \rightarrow \frac{dP}{dt} = kP$

$= 0.014P(30)$

$= 0.014(760)$

$= \boxed{11}$ people/year

Solution #2: let $A(t)$ be the grams of salt in the tank at time, t.

$\frac{dA}{dt}$ = rate in - rate out

rate in = concentration of salt in inflow X input rate of Brine

$= (1\,g/L)(4\,L/min)$

$= 4\,g/min$

rate out = concentration of salt in inflow X output rate of Brine

$= \left(\frac{A(t)}{200}\,g/L\right)(4\,L/m)$

$= \frac{A(t)}{50}\,g/min$

Kate Jordan - Section 1

Solution #2 continued:

$$\frac{dA}{dt} = \text{rate in} - \text{rate out}$$

$$= 4 - \frac{A(t)}{50}$$

$$\frac{dA}{dt} + \frac{1}{50} A(t) = 4$$ → linear equation, use integrating factor method

$$IF = e^{\int \frac{1}{50} dt} = e^{t/50}$$

$$A(t) e^{t/50} = \int 4e^{t/50} dt + C$$

$$= \frac{4}{\frac{1}{50}} e^{t/50} + C$$

$$= 200 e^{t/50} + C$$

$$A(t) = 200 + Ce^{-t/50}$$

at $t = \emptyset$, $A = 30$ g

$$\Rightarrow A(\emptyset) = 30 = 200 + Ce^{0}$$

$$-170 = C$$

$$\boxed{A(t) = 200 - 170e^{-t/50}}$$

Homogeneous / Non-Homogeneous

Legend Matautia

3.1

1) $4y'' - 4y' - 3y = 0$

aux

$4m^2 - 4m - 3 = 0$

$(2m-3)(2m+1) = 0$

$m = -\frac{1}{2}, \frac{3}{2}$

$$\boxed{y = C_1 e^{\frac{-x}{2}} + C_2 e^{\frac{3x}{2}}}$$

Homogeneous / Non-Homogeneous

Legend Mataautia

3.1

2) $4y'' - 12y' + 9y = 45x - 78$

Find y_h

$4y'' - 12y' + 9y = 0$

aux

$4m^2 - 12m + 9 = 0$

$(2m-3)^2 = 0$

$m = \frac{3}{2}$

$y_h = C_1 e^{\frac{3x}{2}} + C_2 x e^{\frac{3x}{2}}$

Find y_p

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$0 - 12A + 9Ax + 9B = 45x - 78$

$9Ax + (9B - 12A) = 45x - 78$

$9A = 45$

$A = 5$

$9B - 12A = -78$

$9B - 60 = -78$

$9B = -18$

$B = -2$

$y_p = 5x - 2$

$$\boxed{y = C_1 e^{\frac{3x}{2}} + C_2 x e^{\frac{3x}{2}} + 5x - 2}$$

Maria Rossman $\underline{3.1}$ Solution #1.)

$$y = C_1 x + C_2 \ln x$$

$$\frac{dy}{dx} = y' = C_1 + C_2 (\ln x + 1)$$

$$y' = C_1 + C_2 + C_2 \ln x$$

With $y(1) = 3$,

$$C_1(1) + C_2(1)\ln(1) = 3$$

$$C_1 = 3$$

With $y'(1) = -1$

$$C_1 + C_2 + C_2 \ln(1) = -1$$

$$C_1 + C_2 = -1$$

$$3 + C_2 = -1$$

$$C_2 = -4$$

Substituting into general solution:

$$\boxed{y = 3x - 4\ln x}$$

Maria Rossman $\underline{3.1}$ Solution #2.)

$f_1(x) = 5,\ f_2(x) = \cos^2 x,\ f_3(x) = \sin^2 x$

linearly dependent if $W(f_1, f_2, f_3) \neq 0$

$$W(f_1, f_2, f_3) = \begin{vmatrix} 5 & \cos^2 x & \sin^2 x \\ 0 & -2\sin x \cos x & 2\sin x \cos x \\ 0 & -2\cos 2x & 2\cos 2x \end{vmatrix}$$

$$= 5\begin{vmatrix} -2\sin x \cos x & 2\sin x \cos x \\ -2\cos 2x & 2\cos 2x \end{vmatrix} - \cos^2 x |0| + \sin^2 x |0|$$

$$= 5(-2\sin 2x \cos 2x + 2\sin 2x \cos 2x) - 0 + 0$$

$$= 5(0) - 0 + 0$$

$$= 0 - 0 + 0$$

$\boxed{W(f_1, f_2, f_3) = 0}$

therefore linearly dependent.

Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Form the general solution.

① $y'' - y' - 12y = 0$; e^{-3x}, e^{4x} $(-\infty, \infty)$

② $y'' - 2y' + 5y = 0$; $e^{x}\cos 2x$, $e^{x}\sin 2x$ $(-\infty, \infty)$

3.1 Fund. Set of Solutions - Maryana Castro

SOLUTION 3.3 Real Roots

$$y''' + 3y'' + 3y' + y = 0$$

Auxiliary Equation

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$(m+1)(m^2 + 2m + 1) = 0$$

$$(m+1)(m+1)(m+1) = 0$$

$$m_1 = -1 \quad m_2 = -1 \quad m_3 = -1$$

$$y = C_1 e^{-x} + C_2 e^{-x} + C_3 e^{-x}$$

or

$$y = e^{-x}(C_1 + C_2 + C_3)$$

mauryn Toole

SOLUTION: 3.3 Real Roots

$$4y'' - 4y' - 3y = 0 \qquad y(0) = 1 \qquad y'(0) = 5$$

Auxiliary Equation

$$4m^2 - 4m - 3 = 0$$

use quadratic formula

$$\frac{4 \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)} \qquad m_1 = \frac{-1}{2} \qquad m_2 = \frac{3}{2}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} \longrightarrow y = C_1 e^{\frac{-x}{2}} + C_2 e^{\frac{3x}{2}}$$

solve for C_1 and C_2

$$y' = \frac{-1}{2} C_1 e^{\frac{-x}{2}} + \frac{3}{2} C_2 e^{\frac{3x}{2}}$$

1 $y(0) = 1$: $1 = C_1 e^0 + C_2 e^0$

$$1 = C_1 + C_2$$

$y'(0) = 5$: $5 = \frac{-1}{2} C_1 e^0 + \frac{3}{2} C_2 e^0$

2 $5 = \frac{-1}{2} C_1 + \frac{3}{2} C_2$

multiply by 2

$$10 = -C_1 + 3C_2$$

3 $C_1 = 1 - C_2$

$$10 = -(1 - C_2) + 3C_2$$

$$10 = -1 + C_2 + 3C_2$$

$$11 = 4C_2$$

$$C_2 = \frac{11}{4}$$

4 $1 - \frac{11}{4} = C_1$

$$C_1 = \frac{-7}{4}$$

$$y = \frac{-7}{4} e^{\frac{-x}{2}} + \frac{11}{4} e^{\frac{3x}{2}}$$

mauryn Toole

3.3-complex roots (3.3 #11) [solution] Maya Gonzales

#1 Find the general solution of the given second-order differential equation:

$$y'' - 4y' + 5y = 0$$

$$m^2 e^{mx} - 4me^{mx} + 5e^{mx} = 0$$

$$e^{mx}(m^2 - 4m + 5) = 0$$

$$m^2 - 4m + 5 = 0$$

$a = 1$
$b = -4$
$c = 5$

$$\frac{+(+4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{4 \pm \sqrt{16 - 20}}{2}$$

$$\frac{4 \pm \sqrt{-4}}{2} \rightarrow \frac{4 \pm i\sqrt{4}}{2}$$

$$\frac{4 \pm 2i}{2} \rightarrow m = 2 \pm i$$

$$\boxed{y = e^{2x}(c_1 \cos x + c_2 \sin x)}$$

Step 1: sub in your known values of y, y', and y''

* remember:
$y = e^{mx}$
$y' = me^{mx}$
$y'' = m^2 e^{mx}$

Step 2: factor out e^{mx}.
↳ $e^{mx} \neq 0 \rightarrow$ cancels out

Step 3: X can't factor, use quadratic →
↳ solve

* remember quadratic form.
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 4: plug into equation:
$\alpha \pm \beta i$
alpha Beta

*general solution:

$$y = e^{\alpha x}[c_1 \cos(\beta x) + c_2 \sin(\beta x)]$$

3.3 Complex roots (3.3 #39) [Solution] Maya Gonzales

#2. BVP

$$y'' + y = 0 \qquad y(0) = 0$$
$$y'(\pi/2) = 0$$

$$e^{mx}(m^2+1) = 0$$

$$m^2 + 1 = 0 \rightarrow m^2 = -1$$

$$m = \sqrt{-1} = \pm i$$

$$y = c_1 \cos x + c_2 \sin x \quad (1)$$

$$y' = \frac{d}{dx}(c_1 \cos x + c_2 \sin x)$$
$$= -c_1 \sin x + c_2 \cos x$$

$$y(0) = 0 \rightarrow c_1 \underbrace{\cos(0)}_{1} + c_2 \underbrace{\sin(0)}_{0} = 0$$

$$c_1 = 0$$

Step 1: Solve auxiliary equation

* remember
$y = e^{mx}$
$y' = me^{mx}$
$y'' = m^2 e^{mx}$

Step 2: plug into eq. to find the general solution.

Step 3: calculate the derivative $\rightarrow y'$

Step 4: plug in initial values.

Step 5: plug values for c_1 and c_2 into the first general solution.

$$y'(\pi/2) = 0 \rightarrow -c_1 \cos(\pi/2) + c_2 \sin(\pi/2) = 0$$

$$c_2 = 0$$

$$y = 0 \cos(0) + 0 \sin x = 0$$

general solution $\rightarrow \boxed{y = 0}$ (2)

* if c_1 or $c_2 \neq 0$, then you would plug in the #s to the first general solution for c_1 and c_2 and get an equation.

36

Mikaela Morris

$$4x^2y'' + y = 0$$

$$y = x^m$$

$$y = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$4x^2y'' + y = 4x^2 m(m-1)x^{m-2} + x^m = 0$$

$$[4m(m-1)+1]x^m = 0$$

$$[4m^2 - 4m + 1]x^m = 0$$

$$4m^2 - 4m + 1 = 0$$

$$(2m-1)(2m-1) = 0$$

$$2m = 1$$

$$m = 1/2$$

$$\boxed{y = c_1 x^{1/2} + c_2 x^{1/2} \ln x}$$

3.6

Mikaela Morris

$$x^3y''' - 6y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)x^{m-3}$$

$$x^3y''' - 6y = x^3 m(m-1)(m-2)x^{m-3} - 6(x^m) = 0$$

$$m(m-1)(m-2)x^m - 6x^m = 0$$

$$[m(m-1)(m-2) - 6]x^m = 0$$

$$[m(m^2 - 2m - m + 2) - 6]x^m = 0$$

$$(m^3 - 3m^2 + 2m - 6)x^m = 0$$

$$m^3 - 3m^2 + 2m - 6 = 0$$

$$(m-3)(m^2+2) = 0$$

$$(m-3)(m - i\sqrt{2})(m + i\sqrt{2}) = 0$$

$$m = 3 \quad m = i\sqrt{2} \quad m = -i\sqrt{2}$$

$$\boxed{y = C_1x^3 + C_2\cos(\sqrt{2}\ln x) + C_3\sin(\sqrt{2}\ln x)}$$

Section 3.6

Adam Bretsch
Math 310
12-7-21

14. $x^2y'' - 7xy' + 41y = 0$

$y = x^m$, $y' = mx^{m-1}$, $y'' = (m^2-m)x^{m-2}$

$$x^2y'' - 7xy' + 41y = x^2(m^2-m)x^{m-2} - 7xmx^{m-1} + 41x^m$$

$$= (m^2-m)x^m - 7mx^m + 41x^m$$

$$= x^m(m^2-m-7m+41)$$

$$= x^m(m^2-8m+41)$$

$m^2 - 8m + 41 = 0$

$$m = \frac{8 \pm \sqrt{64-164}}{2} = \frac{8 \pm \sqrt{-100}}{2} = \frac{8 \pm 10i}{2} = 4 \pm 5i$$

$\alpha \pm \beta i = 4 \pm 5i$

$$\boxed{y = x^4\left[c_1 \cos(5\ln x) + c_2 \sin(5\ln x)\right]}$$

Adam Bretsch
Math 310
12-7-21

Section 3.6

37. $y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$

$y = x^{\alpha}\left[c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x)\right]$

$\alpha = 0, \quad \beta = 1 \quad \Rightarrow \quad m_1 = i, \; m_2 = -i$

$\therefore (m - i)(m + i) = 0$

$m^2 + 1 = 0$

$ax^2y'' + bxy' + cy = 0$

$am^2 + (b - a)m + c = 0$

$a = 1, \quad b = 1, \quad c = 1$

DE: $\boxed{x^2y'' + xy' + y = 0}$

Solutions

1. From equation $m\frac{d^2x}{dt^2} = -kx$ we get $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

With $x'' + \omega^2 x = 0$ we can get $\omega = \sqrt{\frac{k}{m}}$

So $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$k = 4\pi^2 f^2 m$

$= 4\pi^2 \cdot \left(\frac{2}{\pi}\right)^2 \cdot 20$

$\boxed{k = 320\ N/m}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{320}{80}}$

$\boxed{f = \frac{1}{\pi}\ \text{cycles/s}}$

2. $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{E(t)}{L}$

$L = 0.05\ h,\ R = 2\ \Omega,\ C = 0.01\ f,\ E(t) = 0\ V$

$\frac{d^2q}{dt^2} + 40\frac{dq}{dt} + 2000q = 0$

Auxillary equation: $m^2 + 40m + 2000 = 0$

Using the quadratic formula, we get: $m_1 = -20 - 40i$, $m_2 = -20 + 40i$

$q(t) = e^{-20t}(c_1\cos 40t + c_2\sin 40t)$

Use conditions $q(0) = 5$ and $q'(0) = 0$ since $q'(0) = i(0)$

$q(t) = e^{-20t}(5\cos 40t + 2.5\sin 40t)$

At $t = 0.01\,s$ $q(0.01) = e^{-20(0.01)}(5\cos 40(0.01) + 2.5\sin 40(0.01))$

$\boxed{q(0.01) = 4.568\ C}$

Charge is equal to 0 when $q(t) = 0$

$e^{-20t}(5\cos 40t + 2.5\sin 40t) = 0$

$5\cos 40t + 2.5\sin 40t = 0$

$\tan 40t = -2$

$40t = -1.10715 + k\pi$

$t = -0.02768 + \frac{k\pi}{40}$

First time when time is positive is $k = 1$

$\boxed{t = 0.05086\ s}$

Ryan Young

3.8 Mass/Spring & LRC-Circuits Solutions

1. $m\frac{d^2x}{dt^2} = -kx + f(t)$

$\frac{d^2x}{dt^2} + \frac{k}{m}x = \frac{1}{m}f(t)$

$\frac{d^2x}{dt^2} + 16x = 34e^{-2t}\cos 4t$

$x'' + 16x = 0$

$m^2 + 16 = 0$

$m_1 = -4i \quad m_2 = +4i$

$x_c(t) = c_1\cos 4t + c_2 \sin 4t$

$x_p(t) = e^{-2t}(A\cos 4t + B\sin 4t)$

$x'_p(t) = e^{-2t}[(-2A + 4B)\cos 4t) + (-4A - 2B)\sin 4t]$ } put into DEq

$x''_p(t) = e^{-2t}[(-12A - 16B)\cos 4t + (16A - 12B)\sin 4t]$ }

$e^{-2t}[(4A - 16B)\cos 4t + (16A + 4B)\sin 4t] = 34e^{-2}\cos 4t$

$4A - 16B = 34, \quad 16A + 4B = 0$

$A = \frac{1}{2}, \quad B = -2$

$x_p(t) = e^{-2t}(\frac{1}{2}\cos 4t - 2\sin 4t)$

$x(t) = c_1\cos 4t + c_2\sin 4t + e^{-2t}(\frac{1}{2}\cos 4t - 2\sin 4t)$

* Plug in initial conditions to find c_1 & c_2 * $(x(0) = 0, x'(0) = 0)$

$\boxed{x(t) = -\frac{1}{2}\cos 4t + \frac{9}{4}\sin 4t + e^{-2t}(\frac{1}{2}\cos 4t - 2\sin 4t)}$

2. $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{E(t)}{L}$ * Plug in given values

$\frac{d^2q}{dt} + 40\frac{dq}{dt} + 2000q = 0$

$m^2 + 40m + 2000 = 0 \Rightarrow m_1 = -20 - 40i, m_2 = -20 + 40i$

$q(t) = e^{-20t}(c_1\cos 40t + c_2\sin 40t)$ * initial conditions *

$q(t) = e^{-20t}(5\cos 40t + 2.5\sin 40t)$

$\boxed{q(0.01s) = 4.568C}$

* Charge = 0 : set $q(t) = 0$, solve for t: $\boxed{t = 0.05086s}$

SHAELYN TAVARES

CH 4.2 REVIEW PROBLEMS SOLUTIONS

1) FIND $L^{-1}\left[\dfrac{1}{S(S^2+1)}\right]$

$$\frac{1}{S(S^2+1)} = \frac{1}{S}F(S) \longrightarrow \begin{array}{l} F(S) = \dfrac{1}{S^2+1} \\ f(X) = L^{-1}[F(S)] = \text{SIN}X \end{array}$$

$$L^{-1}\left[\frac{1}{S(S^2+1)}\right] = L^{-1}\left[\frac{1}{S}F(S)\right] = \int_0^X f(t)\,dt$$

$$= \int_0^X \text{SIN}\,t\,dt = 1-\text{COS}\,X$$

SHAELYN TAVARES

2) SOLVE THE FOLLOWING IVP USING LAPLACE TRANSFORM:

$$y'' - 6y' + 9y = t \;;\; y(0)=0 \quad y'(0)=1$$

$$L[y''] - 6L[y'] + 9L[y] = L[t]$$

$$\Leftrightarrow \; s^2Y(s) - sy(0) - y'(0) - 6sY(s) - 6y(0) + 9Y(s) = \frac{1}{s^2}$$

$$\Leftrightarrow \; s^2Y(s) - 1 - 6sY(s) + 9Y(s) = \frac{1}{s^2} \;\Leftrightarrow\; (s^2 - 6s + 9)Y(s) = \frac{1}{s^2} + 1$$

$$\Leftrightarrow \; (s-3)^2 Y(s) = \frac{s^2+1}{s^2} \;\Leftrightarrow\; Y(s) = \frac{(s^2+1)}{s^2(s-3)^2}$$

$$L^{-1}\left[\frac{s^2+1}{s^2(s-3)^2}\right] \longrightarrow \frac{As+B}{s^2} + \frac{Cs+D}{(s-3)^2}$$

$$s^2+1 = (As+B)(s-3)^2 + (Cs+D)(s^2)$$

$$= As^3 - 6As^2 + 9As + Bs^2 - 6Bs^2 - 6Bs + 9B + Cs^3 + Ds^2$$

$$= (A+C)s^3 + (B+D-6A)s^2 + (9A-6B)s + 9B$$

$$1 = 9B \longrightarrow B = \frac{1}{9}$$

$$s: \quad 0 = 9A - 6B \longrightarrow A = \frac{2}{3}, \; B = \frac{2}{27}$$

$$s^2: \quad 1 = B + D - 6A \longrightarrow D = 6A - B + 1 = 6\cdot\frac{2}{27} - \frac{1}{9} + 1$$

$$s^3: \quad 0 = A + C \longrightarrow C = (-1)A = -2/27$$

$$\Leftrightarrow \; Y(s) = \frac{1}{27}\left[\frac{2s+3}{s^2} - \frac{2s-36}{(s-3)^2}\right] \;\Leftrightarrow\; Y(s) = \frac{1}{27}\left[\frac{2}{s} + \frac{3}{s^2} - 2\frac{(s-3)-15}{(s-3)^2}\right]$$

$$\Leftrightarrow \; Y(s) = \frac{1}{27}\left[\frac{2}{s} + \frac{3}{s^2} - 2\frac{1}{(s-3)} + 30\frac{1}{(s-3)^2}\right]$$

$$\Leftrightarrow \; y(t) = \frac{1}{27}\left[2L^{-1}\left[\frac{1}{s}\right] + 3L^{-1}\left[\frac{1}{s^2}\right] - 2L^{-1}\left[\frac{1}{s-3}\right] + 30L^{-1}\left[\frac{1}{(s-3)^2}\right]\right]$$

$$\Leftrightarrow \; y(t) = \frac{1}{27}(2 + 3t - 2e^{3t} + 30te^{3t})$$

$$y(t) = \frac{2}{27} + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$$

Sydney Brenneman

Section 4.3 Translation Theorems: Translation on s-axis ANSWER (1)

(1) $\mathcal{L}^{-1}\left\{\dfrac{s/2 + 5/3}{s^2+4s+6}\right\}$

$$\frac{s/2 + 5/3}{s^2+4s+6} = \frac{s/2 + 5/3}{(s+2)^2+2} = \frac{(1/2)(s+2) + 2/3}{(s+2)^2+2}$$

$$= \frac{1}{2}\,\frac{s+2}{(s+2)^2+2} + \frac{2}{3}\,\frac{1}{(s+2)^2+2}$$

$$\mathcal{L}^{-1}\left\{\frac{s/2 + 5/3}{s^2+4s+6}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+2}\right\} + \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+2}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}\bigg|_{s \to s+2}\right\} + \frac{2}{3\sqrt{2}}\mathcal{L}^{-1}\left\{\frac{\sqrt{2}}{s^2+2}\bigg|_{s \to s+2}\right\}$$

$$= \frac{1}{2}e^{-2t}\cos\sqrt{2}t + \frac{\sqrt{2}}{3}e^{-2t}\sin\sqrt{2}t$$

Sydney Brenneman

Section 4.3 Translation Theorems: Translation on s-axis ANSWER 2

2 Solve $y'' + 4y' + 6y = 1 + e^{-t}$, $y(0) = 0$, $y'(0) = 0$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{1\} + \mathcal{L}\{e^{-t}\}$$

$$s^2Y(s) - sy(0) - y'(0) + 4[sY(s) - y(0)] + 6Y(s) = \frac{1}{s} + \frac{1}{s+1}$$

$$(s^2 + 4s + 6)Y(s) = \frac{2s+1}{s(s+1)}$$

$$Y(s) = \frac{2s+1}{s(s+1)(s^2+4s+6)}$$

$$Y(s) = \frac{1/6}{s} + \frac{1/3}{s+1} - \frac{s/2 + 5/3}{s^2+4s+6}$$

$$y(t) = \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+2}\right\}$$

$$- \frac{2}{3\sqrt{2}}\mathcal{L}^{-1}\left\{\frac{\sqrt{2}}{(s+2)^2+2}\right\}$$

$$= \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}e^{-2t}\cos\sqrt{2}t - \frac{\sqrt{2}}{3}e^{-2t}\sin\sqrt{2}t$$

Will Freking — 4.3 Translation on the t-axis

#69. ① $y'' + y = f(t)$ $\quad y(0)=0$, $y'(0)=1$ $\quad f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$

We know, $\quad \rightarrow f(t) = \begin{cases} 0 & 0 \le t < a \\ g(t) & a \le t < b \\ 0 & t \ge b \end{cases}$

$f(t) = g(t)[u(t-a) - u(t-b)]$

$g(t)=1 \quad a=\pi \quad b=2\pi$

② $f(t) = 1\,[u(t-\pi) - u(t-2\pi)]$

$= u(t-\pi) - u(t-2\pi)$

③ $y'' + y = u(t-\pi) - u(t-2\pi)$ $\longrightarrow$ ④ $L(y''+y) = L(u(t-\pi) - u(t-2\pi))$

$L(y'') + L(y) = L(u(t-\pi)) - L(u(t-2\pi))$

$s^2Y(s) - s\,y(0) - y'(0) + Y(s) = L(u(t-\pi)) - L(u(t-2\pi))$

⑤ $L(f(t-a)u(t-a)) = e^{-as} L(f(t))$ ★

$e^{-\pi s} L(1) - e^{-2\pi s} L(1)$

$\downarrow \frac{1}{s} \qquad \downarrow \frac{1}{s}$

$\frac{e^{-\pi s}}{s} \qquad \frac{e^{-2\pi s}}{s}$

→ after inserting y(0) and y'(0)!

⑥ $s^2Y(s) - s(0) - 1 + Y(s) = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi}}{s}$

$\frac{(s^2+1)Y(s) - 1 + 1}{(s^2+1)} = \frac{\frac{e^{-\pi s}}{s} - \frac{e^{-2\pi}}{s} + 1}{(s^2+1)}$

⑦ $Y(s) = \frac{1}{(s^2+1)} + \frac{e^{-\pi s}}{s(s^2+1)} - \frac{e^{-2\pi s}}{s(s^2+1)}$

⑧ $\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \rightarrow 1 = A(s^2+1) + (Bs+C)(s)$

$As^2 + Bs^2 = 0$

$\underline{s=0}$ $\underline{A=1}$ $\quad A+B=0$, $1+B$, $\underline{B=-1}$

$C=0$ because there are no coefficients of s

⑨ $\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$

⑩ $Y(s) = \frac{1}{(s^2+1)} + e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+1}\right) - e^{-2\pi s}\left(\frac{1}{s} - \frac{s}{s^2+1}\right)$

$= \frac{1}{(s^2+1)} + \frac{e^{-\pi s}}{s} - \frac{se^{-\pi s}}{(s^2+1)} - \frac{e^{-2\pi s}}{s} - \frac{se^{-2\pi s}}{(s^2+1)}$

⑪ $y(t) = L^{-1}\left(\frac{1}{s^2+1}\right) + L^{-1}\left(\frac{e^{-\pi s}}{s}\right) - L^{-1}\left(\frac{se^{-\pi s}}{(s^2+1)}\right) - L^{-1}\left(\frac{e^{-2\pi s}}{s}\right) - L^{-1}\left(\frac{se^{-2\pi s}}{(s^2+1)}\right)$

★ $L^{-1}(e^{-as}F(s)) = f(t-a)u(t-a)$, $\quad L^{-1}\left(\frac{1}{s}\right) = 1$, $\quad L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin(at)$

$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos(at)$

⑫ $y(t) = \sin(t) + 1\cdot u(t-\pi) - \cos(t-\pi)u(t-\pi) - 1\cdot u(t-2\pi) + \cos(t-2\pi)u(t-2\pi)$

★ simplify

$\boxed{y(t) = \sin(t) + [1 - \cos(t-\pi)]\,u(t-\pi) - [1 - \cos(t-2\pi)]u(t-2\pi)}$

Steven Iannicca

Section 2 - Math 310

4.4 Transforms of Integrals

(19.) $f(t) = 4t,\ g(t) = 3t^2$

$\Rightarrow$ use convolution definition & theorem 4.1.1

$$\Rightarrow 4t * 3t^2 = \int_0^t 4(t-\tau)3\tau^2 d\tau \quad \text{(convolution def.)}$$

$$\Rightarrow 12t\int_0^t \tau^2 d\tau - 12 \cdot \frac{t^4}{4}$$

$$\Rightarrow t^4$$

$$\Rightarrow \mathscr{L}\{4t * 3t^2\} = \mathscr{L}\{t^4\},\ n = 4 \quad \text{(theorem 4.1.1)}$$

$$\Rightarrow \mathscr{L}\{4t * 3t^2\} = \frac{4!}{s^5} = \frac{24}{s^5}$$

(33.) $\mathscr{L}\left\{t\int_0^t \sin\tau\, d\tau\right\}$

$\Rightarrow$ use convolution definition

$$\Rightarrow \mathscr{L}\left\{t\int_0^t \sin\tau\, d\tau\right\} = \mathscr{L}\{t \cdot (1 * \sin t)\}$$

$$\Rightarrow -\frac{d}{ds}\mathscr{L}\{1 * \sin t\}$$

$$\Rightarrow -\frac{d}{ds}\left(\mathscr{L}\{1\}\mathscr{L}\{\sin t\}\right)$$

$$\Rightarrow -\frac{d}{ds}\left(\frac{1}{s(s^2+1)}\right)$$

$$\Rightarrow \mathscr{L}\left\{t\int_0^t \sin\tau\, d\tau\right\} = \frac{3s^2+1}{s^2(s^2+1)^2}$$

Will Lawler
4.4
Derivatives of Transforms

24 $\mathcal{L}(t^2 * te^t)$

$= \mathcal{L}(t^2) \cdot \mathcal{L}(te^t)$

$= \frac{2!}{s^3}\left(-\frac{d}{ds}\left(\frac{1}{s-1}\right)\right) = \left(\frac{2}{s^3}\right) \cdot -\frac{d}{ds}(s-1)^{-1} = \frac{2}{s^3} \cdot -\left(\frac{1}{(s-1)^2}\right)(-1)$

$= \frac{2}{s^3(s-1)^2}$

4.4
Will Lawlor
Derivatives of Transforms

12 $y'' + y = \sin t \qquad y(0) = 1 \qquad y'(0) = -1$

$$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(\sin t)$$

$$s^2 Y(s) - s\,y(0) - y'(0) + Y(s) = \frac{1}{s^2+1}$$

$$(s^2+1)\,Y(s) - s + 1 = \frac{1}{s^2+1}$$

$$(s^2+1)\,Y(s) = s - 1 + \frac{1}{(s^2+1)}$$

$$Y(s) = \frac{s}{s^2+1} - \frac{1}{s^2+1} + \frac{1}{(s^2+1)^2}$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) + \mathcal{L}\left(\frac{1}{(s^2+1)^2}\right)$$

$$\mathcal{L}\left(\frac{2k^3}{(s^2+k^2)^2}\right) = \sin kt - kt\cos kt, \quad k = 1$$

$$\mathcal{L}^{-1}(Y(s)) = \cos t - \sin t + \frac{1}{2}\mathcal{L}\left(\frac{2(1)^3}{(s^2+1^2)^2}\right) = \cos t - \sin t + \frac{(\sin t - t\cos t)}{2}$$

$$= \frac{-\sin t}{2} + \cos t - \frac{t\cos t}{2}$$